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BFKL evolution and universal structure function at very small x.

N.N. Nikolaev[a,b], and B.G. Zakharov[b]

[a] *IKP(Theorie), KFA Jülich, 5170 Jülich, Germany*

[b] *L. D. Landau Institute for Theoretical Physics, GSP-1, 117940, ul. Kosygina 2, Moscow 117334, Russia.*

Abstract

The Balitskii-Fadin-Kuraev-Lipatov (BFKL) and the Gribov-Lipatov-Dokshitzer-Altarelli-Parisi (GLDAP) evolution equations for the diffractive deep inelastic scattering at $\frac{1}{x} \gg 1$ are shown to have a common solution in the weak coupling limit: $F_2(x, Q^2) \propto [\alpha_S(Q^2)]^{-\gamma} \left(\frac{1}{x}\right)^{\Delta_{\mathbf{IP}}}$. The exponent γ and the pomeron intercept $\Delta_{\mathbf{IP}}$ are related by $\gamma\Delta_{\mathbf{IP}} = \frac{4}{3}$ for the $N_f = 3$ active flavors. The existence of this solution implies that there is no real clash between the BFKL and GLDAP descriptions at very small x. We present derivation of this solution in the framework of our generalized BFKL equation for the dipole cross section, discuss conditions for the onset of the universal scaling violations and analyse the pattern of transition from the conventional Double-Leading-Logarithm approximation for the GLDAP evolution to the BFKL evolution at large $\frac{1}{x}$.

E-mail: kph154@zam001.zam.kfa-juelich.de

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1 Introduction.

The asymptotic behavior of diffractive scattering in perturbative QCD is usually discussed in the framework of the BFKL pomeron [1]. Recently, we took advantage of exact diagonalization of the S-matrix of diffractive scattering in terms of the dipole cross section [2,3] and derived the generalized BFKL equation for the perturbative dipole cross section [4-6]. In [4-6] we studied the spectrum of our generalized BFKL equation and determined the intercept of the pomeron and the asymptotic dipole cross section for the rightmost singularity in the complex j-plane.

The central result of the present paper is a derivation of the asymptotic pomeron solution of our generalized BFKL equation,

$$\sigma_{\mathbf{I\!P}}(\xi,r) = \sigma_{\mathbf{I\!P}}(r)\exp(\Delta_{\mathbf{I\!P}}\xi) \propto r^2\left[\frac{1}{\alpha_S(r)}\right]^{\gamma-1}\exp(\Delta_{\mathbf{I\!P}}\xi)\,, \tag{1}$$

where the exponent γ is related to the pomeron intercept $\Delta_{\mathbf{I\!P}}$ by

$$\gamma = \frac{12}{\beta_0\Delta_{\mathbf{I\!P}}}\,. \tag{2}$$

Here $\beta_0 = 11 - \frac{2}{3}N_f = 9$ for $N_f = 3$ active flavours, r is the transverse size of the colour dipole, $\alpha_S(r)$ is the running QCD coupling and $\xi = \log(\frac{x_0}{x})$, where x_0 ∼0.1-0.01 corresponds to the onset of the leading-log($\frac{1}{x}$) approximation. We shall demonstrate that, to an accuracy $\sim \alpha_S(r)$, the new solution (1) is a low-x limit of solutions of the conventional GLDAP equation [7]. This new solution must be contrasted to:

i) The Double-Leading-Logarithm Approximation (DLLA) solution [4] to the GLDAP evolution equation,

$$\sigma_{DLLA}(\xi,r) = \sigma(0,r)\sum_{n=0}\frac{\eta^n}{n!(n+1)!} \sim r^2\alpha_S(r)\log\left[\frac{1}{\alpha_S(r)}\right]\frac{1}{\sqrt{\eta}}\exp(2\sqrt{\eta})\,, \tag{3}$$

where η is the expansion parameter of DLLA, $\eta = \frac{12}{\beta_0}\xi L(r)$ and $L(r) = \log\left[\frac{1}{\alpha_S(r)}\right]$.

ii) The solution of the scaling BFKL equation [1] in the case of fixed α_S,

$$\sigma_{\mathbf{I\!P}}(\xi,r) \propto r\exp(\Delta_{\mathbf{I\!P}}\xi)\,. \tag{4}$$

The conventional DLLA solution (3) summs the leading powers $\xi^n L(r)^n$, our new solution (1) manifestly summs all powers of $L(r)$.

The long going debate in the literature on the clash between, and transition from, the GLDAP to the BFKL regime with rising $\frac{1}{x}$, is centered on a comparison of the DLLA solution (3) and the scaling BFKL solution (4) (for the recent review and references see [8,9]). The existence of our new solution of the generalized BFKL equation has the two major implications: Firstly, there is no real clash between the GLDAP and BFKL evolutions and the GLDAP evolution remains a viable description of deep inelastic scatering at very large $\frac{1}{x}$ in the perturbative regime of $\alpha_S(Q^2) \ll 1$. Secondly, comparison of solutions (1) and (4) shows the dramatic difference between the cases of the running and fixed strong coupling for the small-x behavior (see also below, a discussion of the so-called DLLA identity). We conclude this introduction citing the universal structure function

$$F_2(x,Q^2) \propto \left[\frac{1}{\alpha_S(Q^2)}\right]^{\gamma} \left(\frac{1}{x}\right)^{\Delta_{\mathbf{I\!P}}} , \tag{5}$$

which follows from the dipole cross section (1). Most unfortunately, the onset of this universal regime only is expected beyond the kinematical range of the HERA experiments.

The paper is organized as follows: We begin with the brief review of the lightcone description of deep inelastic scattering in terms of the dipole cross section and present our generalized BFKL equation for the dipole cross section. Then we discuss a transition from from the DLLA solution to GLDAP equation to the BFKL solution. We introduce a new comparison of the GLDAP and BFKL solutions in terms of the so-called DLLA identity, and present a derivation of our new solution (1) and of the universal scaling violations at asymptotically large $\frac{1}{x}$. Finally, we comment on the significance of the consistent use of the running coupling for the pomeron cross section. In the conclusions section we summarize our main results.

In the region of very large $\frac{1}{x}$ the virtual photoabsorption can be viewed as interaction with the target proton (nucleus) of the multipartonic lightcone Fock states ($q\bar{q}$, $q\bar{q}g$...) of the photon, which are formed at large distance $\Delta z \sim \frac{1}{m_p x}$ upstream the target. Consequently, the longitudinal momentum partitions z_i and the transverse separations $\vec{\rho}_i$ of partons are conserved in the scattering process, and the diffractive S-matrix is exactly diagonalized in the $(\vec{\rho}, z)$ representation [2]. Photons do not couple to gluons and the higher $q\bar{q}g_1...g_n$ states are radiatively generated from the $q\bar{q}$ dipole of size $\vec{r}$. In [4] we gave regular procedure for construction of the corresponding multiparton lightcone wave functions and of the multiparton total cross sections.

The Fock states with n gluons give the $\propto \log^n(\frac{1}{x})$ contribution to the total photoabsorption cross section, which can be reabsorbed into the energy dependent dipole cross section

$$\sigma(\xi,r) = \sum_{n=0} \frac{1}{n!}\sigma_n(r)\xi^n\,. \tag{6}$$

The total photoabsorption cross section can be written as an expectation value [2]

$$\sigma_{T,L}(\gamma^* N,\xi,Q^2) = \int_0^1 dz \int d^2\vec{r}\; |\Psi_{T,L}(z,r)|^2 \sigma(\xi,r) \tag{7}$$

over the wave function of the $q\bar{q}$ Fock state. The wave functions of the (T) transverse and (L) longitudinal virtual photon of virtuality Q^2 were derived in [2] and read

$$|\Psi_T(z,r)|^2 = \frac{6\alpha_{em}}{(2\pi)^2}\sum_1^{N_f} Z_f^2\{[z^2+(1-z)^2]\varepsilon^2 K_1(\varepsilon r)^2 + m_f^2 K_0(\varepsilon r)^2\}\;, \tag{8}$$

$$|\Psi_L(z,r)|^2 = \frac{6\alpha_{em}}{(2\pi)^2}\sum_1^{N_f} 4Z_f^2\; Q^2\, z^2(1-z)^2 K_0(\varepsilon r)^2\;, \tag{9}$$

where $K_1(x)$ is the modified Bessel function, $\varepsilon^2 = z(1-z)Q^2 + m_f^2$, m_f is the quark mass and z is the fraction of photon's light-cone momentum carried by one of the quarks of the $q\bar{q}$ pair $(0 < z < 1)$. Then, the structure function is calculated as $F_2(\xi,Q^2) = Q^2\,[\sigma_T+\sigma_L]\,/(4\pi^2\alpha_{em})$. Notice that the dipole cross section $\sigma(\xi,r)$ is universal, only $|\Psi_{T,L}|^2$ depend on Q^2 and m_f^2.

In [4-6] we have shown that $\sigma(\xi,r)$ satisfies the generalized BFKL equation

$$\frac{\partial\sigma(\xi,r)}{\partial\xi} = \mathcal{K}\otimes\sigma(\xi,r)\,, \tag{10}$$

where in terms of the expansion (6) the kernel $\mathcal{K}$ is defined by

$$\sigma_{n+1}(r) = \mathcal{K}\otimes\sigma_n(r) =$$
$$\frac{3}{8\pi^3}\int d^2\vec{\rho}_1\;\mu_G^2 \left| g_S(R_1)K_1(\mu_G\rho_1)\frac{\vec{\rho}_1}{\rho_1} - g_S(R_2)K_1(\mu_G\rho_2)\frac{\vec{\rho}_2}{\rho_2}\right|^2 [\sigma_n(\rho_1)+\sigma_n(\rho_2)-\sigma_n(r)]\;. \tag{11}$$

Here $R_c = 1/\mu_G$ is the correlation radius for perturbative gluons, $R_i = \min\{r,\rho_i\}$, $g_S(r)$ is the effective colour charge,

$$\alpha_S(r) = \frac{g_S(r)^2}{4\pi} = \frac{4\pi}{\beta_0 \log\left(\frac{C^2}{\Lambda_{QCD}^2 r^2}\right)}\,, \tag{12}$$

where $C \approx 1.5$ [2] and we impose the infrared freezing $\alpha_S(r > R_f) = \alpha_S^{(fr)} = 0.8$ [6].

In the BFKL scaling limit of $r, \rho_1 \ll R_c$ and of fixed α_S, the kernel $\mathcal{K}$ becomes independent of R_c, Eq. (10) becomes equivalent to the BFKL equation [1] and has the familiar BFKL eigenfunctions

$$E(\xi, r) = r^{1+2i\nu} \exp[\xi \Delta(i\nu)] \tag{13}$$

with the BFKL eigenvalue (intercept)

$$\Delta(i\nu) = \frac{3\alpha_S}{\pi}[2\Psi(1) - \Psi(\frac{1}{2} - i\nu) - \Psi(\frac{1}{2} + i\nu)]\,, \tag{14}$$

where $\Psi(x)$ is the digamma function.

Deep inelastic scattering at large Q^2 probes $\sigma(\xi, r)$ at small $r^2 \propto 1/Q^2$ ([2-4], see also below). In the DLLA of large but finite ξ and large $L(r)$, the kernel $\mathcal{K}$ is dominated by $r^2 \ll \rho_i^2 \ll R_f^2$, one can neglect $\sigma(r) \ll \sigma(\rho_i)$ and factor out $\alpha_S(r)$ in Eq. (11), which in the DLLA takes on a particularly simple form [4-6]

$$\sigma_{n+1}(r) = \mathcal{K} \otimes \sigma_n(r) = \frac{3r^2\alpha_S(r)}{\pi^2} \int_{r^2}^{R_f^2} \frac{d^2\vec{\rho}}{\rho^4} \sigma_n(\rho)\,, \tag{15}$$

what is equivalent to the large-$\frac{1}{x}$ limit of the GLDAP evolution equation [7]. As a boundary conditon, one can start with the dipole cross section for interaction with the nucleon target [2]

$$\sigma_0(r) = \frac{32}{9} \int \frac{d^2\vec{k}}{(\vec{k}^2 + \mu_G^2)^2} \alpha_S(k^2)\alpha_S(\kappa^2)[1 - G_p(3\vec{k}^2)] \left[1 - \exp(-i\vec{k}\vec{r})\right]\,, \tag{16}$$

where $G_p(q^2)$ is the charge form factor of the proton, $\kappa^2 = \max\{k^2, \frac{C^2}{r^2}\}$. Then, iterations of Eq. (15) produce the familar DLLA solution (3) [4].

The question of whether there is a strong, experimentally observable, difference between the BFKL and GLDAP evolutions (the latter usually being considered to DLLA), is being discussed in the literature for quite a time ([8,9] and references therein). A comparison of the DLLA iterations $\sigma_{n+1}(r) = \mathcal{K} \otimes \sigma_n(r) = 12L(r)\sigma_n(r)/(n+1)\beta_0$ with the BFKL iterations $\sigma_{n+1}(r) = \mathcal{K} \otimes \sigma_n(r) = \Delta_{\mathbf{I\!P}}\sigma_n(r)$ suggests the DLLA breaking at $L(r)/n \approx L(r)/\sqrt{\eta} \lesssim \frac{3}{4}\Delta_{\mathbf{I\!P}}$. (At large η the dominant contribution to the DLLA cross section (3) comes from $n \sim \sqrt{\eta}$.) We can go one step further and compare in detail the ξ-dependence of the solution of our BFKL equation and of the DLLA solution of the GLDAP equation, starting with the identical initial condition Eq. (16). (For the sake of definitness we consider $R_c = 0.275$f which gives $\Delta_{\mathbf{I\!P}} = 0.4$ [6].) Specifically, we compare the effective intercepts $\Delta_{eff}(\xi, r) = \partial \log \sigma(\xi, r)/\partial \xi$ for the two

solutions. Since the DLLA asymptotics (3) works at $\xi \gtrsim 1$, we shall consider the cross section (16) as a result of the GLDAP evolution by ξ_0 units from a lower energy, i.e., we take for the DLLA solution

$$\sigma_{DLLA}(\xi,r) = \sigma_0(r)\sqrt{\frac{\xi_0}{\xi_0+\xi}}\exp\left[2\sqrt{\frac{4}{3}Ł(r)}\left(\sqrt{\xi_0+\xi}-\sqrt{\xi_0}\right)\right]. \tag{17}$$

It satisfies $\sigma_{DLLA}(\xi=0,r)=\sigma_0(r)$ by the construction, and gives the DLLA effective intercept

$$\Delta_{DDLA}(\xi,r) = \sqrt{\frac{4L(r)}{3(\xi_0+\xi)}} - \frac{1}{2(\xi_0+\xi)}\,. \tag{18}$$

The same $\sigma_0(r)$ is taken as the boundary condition for the BFKL equation (10). (Here we assume it to correspond to $x = x_0 \approx 3\cdot 10^{-2}$, the more detailed BFKL phenomenology of deep inelastic scattering will be presented elsewhere [10]).

Firstly we compute $\Delta_{eff}(\xi=0,r)$ from our BFKL equation (10) and make the readjustment

$$L(r) \Longrightarrow \log\left[\frac{\alpha_S^{(fr)}}{\alpha_S(r)}\right] + c \tag{19}$$

such that $\Delta_{DLLA}(\xi=0,r)$ of Eq. (18) gives a good approximation to this $\Delta_{eff}(\xi=0,r)$ at small r. (Recall that $L(r)$ is defined up to an additive constant $c \lesssim 1$.) With $\xi_0 = 1.25$ this is achieved taking $c\approx 0.05$. Secondly, we study how the BFKL and DLLA effective intercepts diverge at large ξ (Fig.1). At $\xi = \log(\frac{x_0}{x}) \sim 1$, both the DLLA and BFKL effective intercepts are smaller than $\Delta_{\mathbf{I\!P}}$ at $r \gtrsim 0.2$f and are larger than $\Delta_{\mathbf{I\!P}} = 0.4$ at smaller r. The good matching of the BFKL and DLLA effective intercepts at small r is not surprizing, since our generalized BFKL equation (10,11) has the GLDAP equation as a limiting case at small r [4-6]. With rising $\frac{1}{x}$, the BFKL effective intercept flattens and tends to $\Delta_{\mathbf{I\!P}}$, rising at large r and decreasing at small r, whereas the DLLA intercept monotonically decreases with ξ at all r, until the DLLA breaking $\Delta_{DLLA}(\xi,r) \leq \Delta_{\mathbf{I\!P}}$ takes place at $\xi=\xi_c(r)$ given by

$$\xi_c(r) = \log\left(\frac{x_0}{x_c(r)}\right) = \frac{4}{3\Delta_{\mathbf{I\!P}}^2}\log\left[\frac{1}{\alpha_S(r)}\right]. \tag{20}$$

(The much discussed boundary suggested in [11] is not born out by our accurate comparison of the DLLA and BFKL solutions.) The intercept $\Delta_{\mathbf{I\!P}}$ is small, $\Delta_{\mathbf{I\!P}} = 0.4$ at $\mu_G = 0.75$GeV [6]. The resulting large numerical factor

$$\frac{4}{3\Delta_{\mathbf{I\!P}}^2} \approx 8\,, \tag{21}$$

which emerges in the r.h.s. of Eq. (20), explains why the close similarity of the BFKL and DLLA solutions persists in such a broad range of r and x, relevant to the HERA experiments.

The dipole cross section $\sigma(\xi, r)$ is related to the more familiar gluon structure function $G(\xi, r)$ by [4,12]

$$G(\xi, r) = xg(x, r) = \frac{3\sigma(\xi, r)}{\pi^2 r^2 \alpha_S(r)}\,, \tag{22}$$

where $g(x, r)$ is the density of gluons at $x = x_0 \exp(-\xi)$ and the virtuality $Q^2 \sim 1/r^2$. In terms of the $G(\xi, r)$ the DLLA equation (15) is equivalent to

$$\kappa(\xi, r) = \frac{\beta_0}{12} \cdot \frac{1}{G(\xi, r)} \frac{\partial^2 G(\xi, r)}{\partial \xi\, \partial L(r)} = 1\,, \tag{23}$$

which we shall refer to as the DLLA identity. One can easily evaluate $\kappa(\xi, r)$ for the experimentally measured gluon distributions. It is interesting to look at the possible departure from the DLLA identity of the above described solution of our generalized BFKL equation (10,11) subject to the boundary condition (16). The results of such a test are shown in Fig.2. We find that our BFKL solution produces $\kappa(\xi, r) \approx 1$ in a very broad range of ξ and r of the practical interest: the DLLA identity holds to the few per cent accuracy at $r \lesssim \frac{1}{3} R_c$, and to the $(20-30)\%$ accuracy even at large r, up to $r^2 \lesssim \frac{1}{2} R_c^2$. The somewhat oscillatory r-dependence of the $\Delta_{eff}(\xi, r)$ is quite natural and has its origin in the contribution of oscillating harmonics with large $|\nu|$ (for instance, see Eq. (13)), which die out at large ξ.

This remarkable finding of $\kappa(\xi, r) \approx 1$ can be understood as follows: In the weak coupling limit we can factor out $\alpha_S(r)$ from the kernel $\mathcal{K}$, and the generalized BFKL equation (10,11) for $G(\xi, r)$ takes the form

$$\frac{\partial G(\xi, r)}{\partial \xi} = \frac{3}{2\pi^2} \int d^2\vec{\rho}_1 \left[\frac{\alpha_S(\rho_1) G(\xi, \rho_1)}{\rho_2^2} + \frac{\alpha_S(\rho_2) G(\xi, \rho_2)}{\rho_1^2} - \frac{r^2 \alpha_S(r) G(\xi, r)}{\rho_1^2 \rho_2^2} \right], \tag{24}$$

in which the leading contribution comes from $\rho_i^2 \gtrsim r^2$. The major function of the term $\propto \alpha_S(r) G(\xi, r)$ is a regularization of the logarithmic singularity at $\rho_{1,2} \to 0$, and at $\alpha_S(r) \ll 1$ this term can be neglected at the expense of the integration cutoff $\rho_{1,2}^2 \gtrsim r^2$. As a result, Eq. (24) takes the form which is identical to the GLDAP equation (15):

$$\frac{\partial G(\xi, r)}{\partial \xi} = \frac{12}{\beta_0} \int_0^{L(r)} dL(\rho)\ G(\xi, r)\,. \tag{25}$$

Now we wish to emphasize that making DLLA is not imperative when solving the GLDAP equation, and evidently Eq. (25) has the one-parametric family of the small-r eigenfunctions

$$G(\gamma,\xi,r)=\exp[\gamma L(r)]\exp(\Delta\xi)=\left[\frac{1}{\alpha_S(r)}\right]^{\gamma}\exp(\Delta\xi)\,, \tag{26}$$

where the exponent γ is related to the intercept Δ by Eq. (2). The corresponding dipole cross section is given by Eq. (1). In contrast to the conventional DLLA solution (26) which only summs the leading powers $[L(r)\xi]^n$, our new solution (26,1) manifestly summs all powers $L(r)^k$. One can easily check that the neglected $\propto \alpha_S(r)G(\xi,r)$ term in Eq. (24) gives the $\propto \alpha_S(r)$ correction to the solution (26). The small-r considerations alone do not fix the intercept Δ and the exponent γ, they are determined from the matching the solution (1) with the large-r solution of our generalized BFKL equation. As we have shown in [6], the value of $\Delta_{\bf I\!P}$ is predominatly controlled by the semiperturbative region of $r \sim R_c$.

In Fig.3a we show the pomeron dipole section found in [6] by a numerical solution of our generalized BFKL equation. In Fig.3b we show that these solutions to a good accuracy satisfy the property

$$\chi=\frac{\sigma_{\bf I\!P}(r)}{r^2}\alpha_S(r)^{\gamma-1}=const\,. \tag{27}$$

Typically, this property holds up to $r \lesssim \frac{1}{2}R_c$. The smaller is the gluon correlation radius R_c, the sooner starts, and the larger becomes with the increasing r, the $\propto \alpha_S(r)$ correction to Eq. (1) (we keep $\alpha_S^{(fr)}=0.8$).

When solving the GLDAP equations exactly (numerically), one of course implicitly summs all powers of $\log[1/\alpha_S(Q^2)]$. The obvious conclusion from the above derivation of (26,1) is that the DLLA solution (3), which only is valid at moderate values of ξ, evolves at larger ξ, beyond the boundary (20), into our new solution (1). Consequently, from the point of view of the practical phenomenology, there is no real clash between the GLDAP and BFKL evolutions. The BFKL evolution is evolution in ξ and requires as the boundary condition the knowledge of $\sigma(\xi=0,r)$ for all r at fixed $\xi=0$. The GLDAP evolution is evolution in $L(r)$, and requires the knowledge of $\sigma(\xi,r_0)$ for all ξ at fixed $r=r_0$. Inspection of Fig. 1 shows that at

$$r=r_0\sim(0.3-0.7)R_c\,, \tag{28}$$

the DLLA and the BFKL intercepts are very close to each other and to the $\Delta_{\bf I\!P}$ in a broad

range of x of the interest for the HERA experiments. Our conclusion is that choosing for the GLDAP evolution the boundary condition at $r_0 \sim 0.15$f, i.e., at $Q_0^2 \sim$ 10-20GeV^2 (for the Q^2-r^2 relationship see below),

$$G(x,Q_0^2),\; xq(x,Q_0^2),\; x\bar{q}(x,Q_0^2) \propto x^{\Delta}\,, \tag{29}$$

one will obtain the GLDAP solutions, which for all the practical purposes will be indistinguishable from the BFKL solution, if Δ is taken equal to $\Delta_{\mathbf{I\!P}}$.

Making use of properties of the modified Bessel functions, after the z-integration one can write

$$\sigma_T(\gamma^* N,\xi,Q^2) = \int_0^1 dz \int d^2\vec{r}\,|\Psi_T(z,r)|^2\sigma(\xi,r)$$
$$\propto \frac{1}{Q^2}\int_{1/Q^2}^{1/m_f^2} \frac{dr^2}{r^2}\frac{\sigma(\xi,r)}{r^2} \propto \frac{1}{Q^2}\int_0^{-\log\alpha_S(Q^2)} dL(\rho)\;G(\xi,\rho) \tag{30}$$

Notice, that the factor $1/Q^2$ in Eq. (30), which provides the Bjorken scaling, comes from the probability of having the $q\bar{q}$ fluctuation of the highly virtual photon. Making use of the pomeron soluiton (26) in (30), we easily obtain the universal asymptotic structure function (5). Since $G(\xi,r)$ slowly rises towards small r, the structure function $F_2(x,Q^2)$ receives a substantial (but not quite dominant) contribution from

$$r^2 \sim \frac{B}{Q^2}\,. \tag{31}$$

The explicit form of the wave function (8) leads to a rather large numerical factor $B \sim 10$ in the relationship (31) (for the related slow onset of the short-distance dominance also see [13]). According to Fig. 1, the BFKL effects are most significant at $r \gtrsim \frac{1}{2}R_c \approx 0.15$f, and the scaling violations in the region of $Q^2 \lesssim$(20-30)GeV^2 seem to be the most interesting ones from the point of view of testing the onset of the BFKL regime.

Eq. (5) predicts the universal, x-independent, scaling violation at small x:

$$-\frac{\partial \log F_2(x,Q^2)}{\partial \log \alpha_S(Q^2)} = \frac{4\pi}{\beta_0\alpha_S(Q^2)}\frac{\partial \log F_2(x,Q^2)}{\partial \log Q^2} = \gamma\,. \tag{32}$$

The exponent γ and the pomeron intercept $\Delta_{\mathbf{I\!P}}$ are not calculable within the perturbative QCD. For the guidance, we cite the results of an analysis [6]: γ = 2.19, 2.81, 3.33, 3.70 for μ_G = 0.3, 0.5, 0.75, 1.0 GeV, respectively. It would have been of great interest to see the universal scaling violation (32) experimentally, but closer inspection of Fig. 1 makes us to conclude

that the applicability region of (32) lies somewhat beyond the kinematical range of the HERA experiments. Indeed, the universal scaling violation comes along with the r-independent universal x-dependence of the structure function (5), i.e, with $\Delta_{eff}(\xi, r) = \Delta_{\bf I\!P}$, whereas Fig. 1 shows that in the kinematic range of HERA, the $\Delta_{eff}(\xi, r)$ for our BFKL solution exhibits a still substantial r-dependence.

The discussion of the BFKL effects in the current literature concentrates upon the approximation of fixed α_S (for the review and references see [8]). Our finding is that the effect of the running coupling constant is quite substantial. Firstly, the pomeron cross section (1) dramatically differs from the $\propto r^1$ BFKL scaling solution (4, 13) for the fixed α_S. Secondly, one can define the counterpart of the DLLA identity for fixed α_S too, making the straightforward substitution

$$L(r) = \frac{\beta_0}{4\pi}\int_{r^2}^{R^2}\frac{d\rho^2}{\rho^2}\alpha_S(\rho) \Longrightarrow \frac{\beta_0}{4\pi}\alpha_S \log\left(\frac{R^2}{r^2}\right) . \tag{33}$$

Then, making use of the BFKL formula for the pomeron intercept [1]

$$\Delta_{\bf I\!P} = \Delta(0) = \frac{12\log 2}{\pi}\alpha_S\,, \tag{34}$$

one readily finds $\kappa = 2\log 2$ for the BFKL pomeron solution (4). This departure from $\kappa = 1$ for the case of the running $\alpha_S(r)$ also emphasizes a dramatic difference between the cases of the fixed and running strong coupling. Therefore, the fixed-α_S considerations are not appropriate for the phenomenology of deep-inelastic scattering.

2 Conclusions

The purpose of this paper has been a comparison of the BFKL and GLDAP evolutions at large $\frac{1}{x}$ in the framework of the dipole cross section description of deep inelastic scattering. We found a new solution (1) which is common to the BFKL equation with the running QCD coupling and to the GLDAP equation, and provides a smooth matching of our generalized BFKL evolution and of the GLDAP evolution at small x. We derived the asymptotic form of the structure function (5) and the universal law Eq. (32) for the scaling violations. Our principle conclusion is that the GLDAP evolution remains a viable phenomenology of the scaling violations at HERA and beyond, provided that one starts with the boundary condition (29) at $Q_0^2 \sim$(10-20) GeV^2.

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Figure captions:

Fig.1 - Comparison of effective intercepts of the DLLA solution (17) and of the solution of our BFKL equation. Both solutions start with the identical dipole cross section at $x = 3 \cdot 10^{-2}$. The pomeron intercept $\Delta_{\mathbf{I\!P}} = 0.4$ is shown by the horizontal line.

Fig.2 - Test of the DLLA identity for the solution of our generalized BFKL equation ($R_c = 0.27$f, $\mu_G = 0.75$GeV).

Fig.3 - The left box: the pomeron dipole cross section $\sigma_{\mathbf{I\!P}}(r)$ for different values of μ_G. The straight lines show the r^1 and r^2 behavior.

The right box: test of the scaling law $\chi = \frac{1}{r^2}\sigma_{\mathbf{I\!P}}(r)[\alpha_S(r)]^{\gamma-1}$ =const.